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Feedback between US and UK Prices: a Frequency Domain Analysis

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Abstract

This paper decomposes the feedback between US and UK price levels by frequency over the period 1791 to 1990. By adapting Geweke's (1982) method of decomposing the feedback between time series to the case of I(1) time series generated by a bivariate error−correction model, we find that most of the feedback between the two time series occurs at very low frequencies. This result provides a reconciliation of the typical rejection of purchasing power parity (PPP) in short−run studies with the findings of paradoxically short half−lives for deviations from PPP often found in long−run studies.

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1. Introduction

The long-run version of the purchasing power parity (PPP) hypothesis holds that, while shocks to nominal exchange rates and prices may result in deviations from PPP at a point in time, eventually PPP will be restored as prices and/or exchange rates respond to the deviation of the real exchange rate from its long-run value. This implies that the (log of the) real exchange rate ought to follow a mean-reverting stochastic process. Tests using very long spans of data, such as that in Lothian and Taylor (1996), typically confirm this implication while those using data from the most recent period of floating exchange rates do not[1]. The latter finding is at odds with the sense of the speed of adjustment toward PPP given by the half-lives of deviations from PPP reported in several studies[2]. In this paper we use an adaptation of the methods proposed by Geweke (1982) to investigate this puzzle.

Geweke's method of decomposing by frequency the feedback, or Granger-causality, between time series enables the researcher to determine at which frequencies one time series causes another so that one may learn, for example, whether the causality from one time series to another represents an influence of the former which is important in the seasonal, business cycle, or permanent evolution of the latter[3]. However, implementation of Geweke's method requires that a finite vector autoregression (VAR) be an accurate approximation to the vector moving average (VMA) representation of the time series. This is not the case for data generated by an error–correction model as the VAR is overdifferenced inducing a unit root in the error term of the VMA and preventing its approximation by a finite VAR. Interest in data generated by error–correction models arises because all cointegrated time series can be represented this way and long-run PPP implies that if (the logs of) the nominal exchange rate and the relevant price levels obey integrated processes (of the same order), they will be cointegrated.

After adapting Geweke's method to the case of $I(1)$ time series generated by a bivariate error–correction model we apply the method to data on US and UK exchange rates and price levels over the period 1791 to 1990. We find that most of the feedback between the two time series occurs at frequencies associated with cycles having periods exceeding 12 years. This finding provides some insight into the rejections of the purchasing power parity (PPP) hypothesis typically found in those studies testing the stationary of the log of the real exchange rate using data from the most recent experience with floating exchange rates. Moreover, it provides a reconciliation of those rejections with the speeds of adjustment implied by half-life calculations - the intuition about adjustment speeds developed by such calculations severely understates the time taken to eliminate deviations from PPP.

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[1] See Froot and Rogoff (1995) for a survey of long-run approaches to testing PPP.

[2] Lothian and Taylor report an estimated half-life of 5.8 years while Abuaf and Jorion (1990) imply a half-life of 3.7 years and Frankel (1990) implies a half-life of 4.1 years.

[3] The phenomenon described by Geweke as “feedback” from one time series to another is precisely that known as Granger-causality from the one to the other. As Pierce (1982) points out, it is not the same usage as in control theory where “feedback” connotes a bidirectional relationship. Throughout this paper we use “feedback” in Geweke's sense so that unidirectional “feedback” is possible. Koo (1996) shows that feedback decomposition may be interpreted as the frequency domain counterpart of variance decomposition in the time domain.

2. Feedback Decomposition in a Bivariate Error–Correction Model

Let $(x_{1,t}, x_{2,t})'$ be a bivariate stochastic process with both $x_{1,t}$ and $x_{2,t}$ being $I(1)$. Suppose that $x_{1,t}$ and $x_{2,t}$ are cointegrated so that there is a number, α, such that $z_t = x_{2,t} - \alpha x_{1,t}$ is $I(0)$. There then exists (Engle and Granger, 1987) an error-correction representation for $(x_{1,t}, x_{2,t})'$ which we write as

$$\begin{pmatrix} \Delta x_{1,t} \\ \Delta x_{2,t} \end{pmatrix} = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} z_{t-1} + \begin{bmatrix} \gamma_{11}(L) & \gamma_{12}(L) \\ \gamma_{21}(L) & \gamma_{22}(L) \end{bmatrix} \begin{pmatrix} \Delta x_{1,t-1} \\ \Delta x_{2,t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{pmatrix} \tag{1}$$

with $\beta_1 \neq 0$ or $\beta_1 \neq 0$ or both. Here Δ is the first difference operator; $\gamma_{kj}(L)$ for $k, j = 1, 2$ are scalar polynomials in the lag operator, L; $E\epsilon_{k,t} = 0$ for $k = 1, 2$; $E\epsilon_{k,t}\epsilon_{j,s} = \sigma_k^2$ for $k = j$ and $s = t$; $E\epsilon_{k,t}\epsilon_{j,s} = \sigma_{kj}$ for $k \neq j$ and $s = t$; and, $E\epsilon_{k,t}\epsilon_{j,s} = 0$ for $k \neq j$ and $s \neq t$. Note that Δx_1 does not Granger-cause Δx_2 if and only if both β_2 and $\gamma_{21}(L)$ are zero.

Equation (1) implies that $(x_{1,t}, x_{2,t})'$ has the VAR representation

$$[(1 - L)(1 - \gamma_{11}(L)L) + \alpha\beta_1 L]x_{1,t} - ((1 - L)\gamma_{12}(L) + \beta_1)Lx_{2,t} = \epsilon_{1,t} \tag{2a}$$

and

$$[(1 - L)(1 - \gamma_{22}(L)L) - \beta_2 L]x_{2,t} - ((1 - L)\gamma_{21}(L) - \alpha\beta_2)Lx_{1,t} = \epsilon_{2,t}. \tag{2b}$$

Subtracting σ_{12}/σ_2^2 times equation (2b) from equation (2a) and rearranging yields

$$\begin{aligned} &\{[(1 - L)(1 - \gamma_{11}(L)L) + \alpha\beta_1 L] + (\sigma_{12}/\sigma_2^2)((1 - L)\gamma_{21}(L) - \alpha\beta_2)L\}x_{1,t} \\ &- \{[((1 - L)\gamma_{12}(L) + \beta_1)L] + (\sigma_{12}/\sigma_2^2)[(1 - L)(1 - \gamma_{22}(L)L) - \beta_2 L]\}x_{2,t} = \eta_{1,t} \end{aligned} \tag{3}$$

where $\eta_{1,t} = \epsilon_{1,t} - (\sigma_{12}/\sigma_2^2)\epsilon_{2,t}$, and has variance $\nu_1^2 = \sigma_1^2 - \sigma_{12}^2/\sigma_2^2$. Observe that $\eta_{1,t}$ is uncorrelated with $\epsilon_{2,t}$ and so is uncorrelated with $x_{2,t}$ as well as with $x_{1,t-\tau}$ and $x_{2,t-\tau}$ for $\tau > 0$. Equation (2b) expresses $x_{2,t}$ as a linear function of $x_{1,t-\tau}$ and $x_{2,t-\tau}$ for $\tau > 0$, while equation (3) expresses $x_{1,t}$ as a linear function of $x_{1,t-\tau}$ for $\tau > 0$ and $x_{2,t-\tau}$ for $\tau \geq 0$. This representation puts all of the instantaneous feedback between x_1 and x_2 into equation (3) while equation (2b) captures only the noninstantaneous feedback from x_1 to x_2.

The feedback from Δx_1 to Δx_2 may be measured by $F_{\Delta x_1 \to \Delta x_2} = \log \kappa_2^2/\sigma_2^2$ where κ_2^2 is the variance of the one-step-ahead error when $\Delta x_{2,t}$ is predicted from its own past. As $\sigma_2^2 = \kappa_2^2$ is necessary and sufficient for Δx_1 not to Granger-cause Δx_2, $F_{\Delta x_1 \to \Delta x_2} > 0$ is necessary and sufficient for Δx_1 to Granger-cause Δx_2. The feedback from Δx_2 to Δx_1, $F_{\Delta x_2 \to \Delta x_1}$, may be measured in an analogous manner. The instantaneous feedback between Δx_1 and Δx_2 may be measured by $F_{\Delta x_1 \cdot \Delta x_2} = \log \sigma_1^2/\nu_1^2 = \log \sigma_1^2\sigma_2^2/\mid \Sigma \mid$ where $\mid \cdot \mid$ denotes the determinant and $\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix}$. The linear dependence between Δx_1 and Δx_2 denoted $F_{\Delta x_1, \Delta x_2}$, is given by the sum of the three linear feedback measures, so that $F_{\Delta x_1, \Delta x_2} = F_{\Delta x_1 \to \Delta x_2} + F_{\Delta x_2 \to \Delta x_1} + F_{\Delta x_1 \cdot \Delta x_2}$.

To decompose the feedback from Δx_1 to Δx_2 by frequency, multiply equation (3) by $((1-L)\gamma_{21}(L)-\alpha\beta_2)L$, use equation (2b) to substitute for $((1-L)\gamma_{21}(L)-\alpha\beta_2)Lx_{1,t}$ and rearrange to give

$$\begin{aligned}\psi(L)\Delta x_{2,t} &= ((1-L)\gamma_{21}(L)-\alpha\beta_2)L\eta_{1,t} \\ &\quad + \{[(1-L)(1-\gamma_{11}(L)L)+\alpha\beta_1 L]+(\sigma_{12}/\sigma_2^2)[((1-L)\gamma_{21}(L)-\alpha\beta_2)L]\}\epsilon_{2,t}\end{aligned}$$

where

$$\begin{aligned}\psi(L) &= (1-L)[(1-\gamma_{22}(L)L)(1-\gamma_{11}(L)L)-\gamma_{12}(L)\gamma_{21}(L)L^2] \\ &\quad - [(1-\gamma_{11}(L)L)+\alpha\gamma_{12}(L)L]\beta_2 L+[\alpha(1-\gamma_{22}(L)L)+\gamma_{21}(L)L]\beta_1 L\end{aligned} \quad . \tag{4}$$

Using the mutual orthogonality of the $\{\eta_{1,t}\}$ and $\{\epsilon_{2,t}\}$ processes, it follows that, $h_{\Delta x_2}(\omega)$, the spectral density of Δx_2, is given by

$$h_{\Delta x_2}(\omega) = \frac{1}{2\pi}\frac{(\sigma_1^2-\sigma_{12}^2/\sigma_2^2)\parallel \xi(\omega)\parallel^2+\sigma_2^2\parallel(\sigma_{12}/\sigma_2^2)\xi(\omega)+\phi(\omega)\parallel^2}{\parallel\psi(e^{-i\omega})\parallel^2}$$

where

$$\xi(\omega) = ((1-e^{-i\omega})\gamma_{21}(e^{-i\omega})-\alpha\beta_2)e^{-i\omega}, \tag{5}$$

$$\phi(\omega) = (1-e^{-i\omega})(1-\gamma_{11}(e^{-i\omega})e^{-i\omega})+\alpha\beta_1 e^{-i\omega} \tag{6}$$

and $\parallel\cdot\parallel$ denotes the modulus[4]. That part of the variance of $\Delta x_{2,t}$ at frequency ω due to variation in that part of $\Delta x_{1,t-\tau}$ for $\tau>0$ uncorrelated with $\Delta x_{2,t-\tau}$ for $\tau>0$ is given by

$$g_{\Delta x_1\to\Delta x_2}(\omega) = \frac{1}{2\pi}\frac{(\sigma_1^2-\sigma_{12}^2/\sigma_2^2)\parallel\xi(\omega)\parallel^2}{\parallel\psi(e^{-i\omega})\parallel^2}. \tag{7}$$

This is the part of the spectral density of Δx_2 due to feedback from Δx_1. The ratio of $g_{\Delta x_1\to\Delta x_2}(\omega)$ to $h_{\Delta x_2}(\omega)$,

$$\mu_{\Delta x_1\to\Delta x_2}(\omega) = \frac{(\sigma_1^2-\sigma_{12}^2/\sigma_2^2)\parallel\xi(\omega)\parallel^2}{(\sigma_1^2-\sigma_{12}^2/\sigma_2^2)\parallel\xi(\omega)\parallel^2+\sigma_2^2\parallel(\sigma_{12}/\sigma_2^2)\xi(\omega)+\phi(\omega)\parallel^2} \tag{8}$$

measures that fraction of the variance of Δx_2 at frequency ω due to feedback from Δx_1. In other words, as Pierce (1982) notes, $\mu_{\Delta x_1\to\Delta x_2}(\omega)$ can be interpreted as the coefficient of determination in the regression of the ω-frequency component of Δx_2 on the ω-frequency component of Δx_1after both have been regressed on lagged values of Δx_2.

[4]The procedure to compute spectral density at frequency ω includes the Fourier transform of the covariance generating function of equation (4). Details can be found in Geweke (1982) and Sargent (1987).

In the case at hand, the measure of the feedback from Δx_1 to Δx_2 at frequency ω defined by Geweke (1982) is given by

$$f_{\Delta x_1 \to \Delta x_2}(\omega) = \log \frac{(\sigma_1^2 - \sigma_{12}^2/\sigma_2^2) \parallel \xi(\omega) \parallel^{\,2} + \sigma_2^2 \parallel (\sigma_{12}/\sigma_2^2)\xi(\omega) + \phi(\omega) \parallel^{\,2}}{\sigma_2^2 \parallel (\sigma_{12}/\sigma_2^2)\xi(\omega) + \phi(\omega) \parallel^{\,2}}. \qquad (9)$$

From equations (8) and (9) we have $\mu_{\Delta x_1 \to \Delta x_2}(\omega) = 1 - e^{-f_{\Delta x_1 \to \Delta x_2}(\omega)}$ so that $g_{\Delta x_1 \to \Delta x_2}(\omega) = h_{\Delta x_2}(\omega)\left[1 - e^{-f_{\Delta x_1 \to \Delta x_2}(\omega)}\right]$. In the next section we analyze the feedback from Δx_1 to Δx_2 using plots of $h_{\Delta x_2}(\omega)$ and $g_{\Delta x_1 \to \Delta x_2}(\omega)$ against ω rather than plots of $f_{\Delta x_1 \to \Delta x_2}(\omega)$ as the latter can become very large at frequencies where the variance of Δx_2 is dominated by the feedback from Δx_1[5]. The feedback from Δx_2 to Δx_1 at frequency ω, can be similarly decomposed by expressions analogous to those developed above for feedback from Δx_1 to Δx_2.

Observe that the statements "$f_{\Delta x_1 \to \Delta x_2}(\omega) = 0$", "$g_{\Delta x_1 \to \Delta x_2}(\omega) = 0$", and "$\mu_{\Delta x_1 \to \Delta x_2}(\omega) = 0$" are equivalent as are their respective converses. From equation (4) it is evident that $\xi(\omega) = 0$ and hence $g_{\Delta x_1 \to \Delta x_2}(\omega) = 0$ if $\beta_2 = 0$ and $\gamma_{21}(e^{-i\omega}) = 0$. The converse is also true[6]. Thus, $g_{\Delta x_1 \to \Delta x_2}(\omega) \neq 0$ implies the presence of feedback from Δx_1 to Δx_2 at frequency ω, while $g_{\Delta x_1 \to \Delta x_2}(\omega) = 0$ implies the absence of feedback at frequency ω.

It is evident from equations (4), (5) and (7) that, as $\omega \to 0$, the feedback from Δx_1 to Δx_2 at frequency ω is dominated by the terms due to the error-correction mechanism because the other parts of $\xi(\omega)$ are $\phi(\omega)$ are eliminated as $1 - e^{-i\omega} \to 0$. In fact, we have that

$$\mu_{\Delta x_1 \to \Delta x_2}(0) = \frac{(\sigma_1^2 - \sigma_{12}^2/\sigma_2^2)\beta_2^2}{\sigma_1^2\beta_2^2 - 2\sigma_{12}\beta_1\beta_2 + \sigma_2^2\beta_1^2}$$

so that, if $\beta_2 = 0$ we have $\mu_{\Delta x_1 \to \Delta x_2}(0) = 0$ regardless of $\gamma_{11}(L)$ or $\gamma_{21}(L)$ provided that $\beta_1 \neq 0$. Moreover, provided Σ is not singular, if $\mu_{\Delta x_1 \to \Delta x_2}(0) = 0$ then $\beta_2 = 0$. If $\beta_1 = 0$ we have $\mu_{\Delta x_1 \to \Delta x_2}(0) = 1 - \sigma_{12}^2/\sigma_1^2\sigma_2^2$ provided that $\beta_2 \neq 0$. As the assumption that x_1 and x_2 are cointegrated implies that at least one of β_1 and β_2 are nonzero, there must be feedback in at least one direction at the zero frequency so that at least one of $\mu_{\Delta x_1 \to \Delta x_2}(0)$ and $\mu_{\Delta x_2 \to \Delta x_1}(0)$ are nonzero. Indeed, this is the only implication of the cointegration of x_1 and x_2 for the feedback between $\Delta x_1$and Δx_2.

3. Analysis of the Feedback between US and UK Prices

In this section we apply the method derived in the previous section to the analysis of the feedback between US and (exchange rate adjusted) UK prices over the period 1791 to 1990 using the data on the US and UK wholesale price indices and the dollar-sterling

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[5]Geweke (1982) shows that $\frac{1}{2\pi}\int_{-\pi}^{\pi} f_{\Delta x_1 \to \Delta x_2}(\omega)d\omega \leq F_{\Delta x_1 \to \Delta x_2}$ with equality if, in this case, the roots of the lag polynomial on $x_{1,t}$ in equation (4) lie outside the unit circle.

[6]If $g_{\Delta x_1 \to \Delta x_2}(\omega) = 0$ with $\gamma_{21}(e^{-i\omega}) \neq 0$ and $\beta_2 \neq 0$ then $\gamma_{21}(e^{-i\omega})(1 - e^{-i\omega}) = \alpha\beta_2$ requires that $\gamma_{21}(e^{-i\omega})$ be the conjugate of $(1 - e^{-i\omega})$, a possibility ruled out by the specification of the model. If $g_{\Delta x_1 \to \Delta x_2}(\omega) = 0$ with $\gamma_{21}(e^{-i\omega}) \neq 0$ and $\beta_2 = 0$ then $\gamma_{21}(e^{-i\omega})(1 - e^{-i\omega}) = 0$ also requires that $\gamma_{21}(e^{-i\omega})$ be the conjugate of $(1 - e^{-i\omega})$ except at $\omega = 0$ but $\beta_2 = 0$ implies $g_{\Delta x_1 \to \Delta x_2}(0) = 0$ anyway. If $g_{\Delta x_1 \to \Delta x_2}(\omega) = 0$ with $\gamma_{21}(e^{-i\omega}) = 0$ and $\beta_2 \neq 0$ then $\gamma_{21}(e^{-i\omega})(1 - e^{-i\omega}) = \alpha\beta_2$ cannot hold.

exchange rate used by Lothian and Taylor (1996). Interest in this feedback arises from its relationship to the long-run version of the PPP hypothesis. This hypothesis holds that, while shocks to nominal exchange rates and prices may result in deviations from PPP at a point in time, eventually PPP will be restored as prices and/or exchange rates respond to the deviation of the real exchange rate from its long-run value. This implies that the (log of the) real exchange rate ought to follow a mean-reverting stochastic process so that, if (the logs of) the nominal exchange rate and the relevant price levels obey integrated processes (of the same order), they will be cointegrated.

To fix ideas, let p_t be the log of the US WPI in year t, p_t^* be the log of the UK WPI in year t, and e_t be the log of the dollar-sterling nominal exchange rate in year t. In the notation of the previous section, define $x_{1,t} = p_t$ and $x_{2,t} = p_t^* + e_t$, the log of the dollar equivalent of the UK WPI, so that, with $\alpha = 1$, $z_t = x_{2,t} - \alpha x_{1,t} = p_t^* + e_t - p_t = \theta_t$ is the log of the dollar-sterling real exchange rate[7].

Lothian and Taylor reject the unit-root hypothesis for θ_t over the period 1791 to 1990 implying that the real exchange rate reverts to its mean eliminating short-run deviations from PPP. ADF tests for p_t and $p_t^* + e_t$ reveal that the unit root hypothesis cannot be rejected for the levels but can be rejected for the first differences[8]. We conclude that the data are consistent with the view that the $(p_t, p_t^* + e_t)'$ process has an error-correction representation. Our estimate of that model is [9]:

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[7]An alternative specification of the model would be to set $x_{1,t} = p_t^* - p_t$ and $x_{2,t} = e_t$, so that, with $\alpha = -1$, $z_t = x_{2,t} - \alpha x_{1,t} = p_t^* + e_t - p_t = \pi_t$ is again the log of the dollar-sterling real exchange rate. With this specification, the feedback at issue is that between Δe_t and $\Delta(p_t^* - p_t)$. This is an arguably more interesting case than that studied here, particularly if purchasing power parity is viewed as (part of) a theory of exchange rate determination. However, interpretation of the empirical results is substantially more problematic than for the specification used here due to the different nominal exchange rate regimes prevailing during the sample period. See Lothian and Taylor (1996) for a discussion of the possible empirical implications of the multiple regimes.

[8]Estimating the model $\Delta x_t = \alpha + \beta t + (\rho - 1)x_{t-1} + \sum_{i=1}^{k-1} \delta_i \Delta x_{t-i} + u_t$ yields t-ratios for the hypothesis that $\rho = 1$ of $-.94$, -10.4, $.01$, -10.1, and -3.63 for p_t, Δp_t, $p_t^* + e_t$, $\Delta(p_t^* + e_t)$, and π_t respectively. Comparison with the 5% critical value of -3.43 from Fuller (1976) produces rejections of the hypothesis for Δp_t, $\Delta(p_t^* + e_t)$, and π_t. For p_t and $p_t^* + e_t$, the t-ratios for the hypothesis that $\beta = 0$ are 1.85 and 1.75 respectively. Comparison with the 5% critical value of 2.79 from Fuller fails to reject the null for either variable. Estimating the model $\Delta x_t = \alpha + (\rho - 1)x_{t-1} + \sum_{i=1}^{k-1} \delta_i \Delta x_{t-i} + u_t$ yields t-ratios for the hypothesis that $\rho = 1$ of .31 and 1.02 for p_t and $p_t^* + e_t$ respectively. Comparison with the 5% critical value of -2.88 from Fuller fails to reject the null for either variable. In each case, the lag length, k, was selected by, beginning with $k = 10$, reducing k by one until the coefficient on Δx_{t-k+1} was significantly different from zero at the 5% level. For the model with the trend term, this method selects ks of 2, 1, 3, 2, and 9 for p_t, Δp_t, $p_t^* + e_t$, $\Delta(p_t^* + e_t)$, and π_t respectively. For the model without the trend term, this method selects ks of 2 and 3 for p_t and $p_t^* + e_t$ respectively.

[9]The numbers beneath each estimated coefficient are absolute t-ratios for the hypothesis that the true value is zero. The lag lengths in the model are chosen by allowing each of the four lag lengths to vary independently from zero to ten in models including the π_{t-1} term in one, or the other, or both equations, and selecting that model minimizing the AIC calculated for the model as a whole. As each equation is thus permitted to have a different set of regressors, each candidate model, as well as the results presented in the text, is estimated by SUR with each equation including a constant term to allow for nonzero means. The AIC is known to have a nonzero asymptotic probability of selecting a lag length longer than the true lag length (Geweke and Meese, 1981). However, the selection criteria studied by Geweke and Meese that asymptotically select the correct lag

$$\Delta p_t = \underset{(1.06)}{.006} + \underset{(5.75)}{.345}\, \Delta(p^*_{t-1} + e_{t-1}) + \epsilon_{1,t}$$

$$\Delta(p^*_t + e_t) = \underset{(3.71)}{.183} - \underset{(3.56)}{.110}\, \theta_{t-1} + \underset{(4.97)}{.341}\, \Delta(p^*_{t-1} + e_{t-1}) - \underset{(2.43)}{.125}\, \Delta(p^*_{t-2} + e_{t-2}) + \epsilon_{2,t}$$

with error covariance matrix

$$\widehat{\Sigma} = \begin{bmatrix} .00666 & .00500 \\ .00500 & .00808 \end{bmatrix}.$$

Note that the lagged real exchange rate term enters only the equation for $\Delta(p^*_t + e_t)$ implying that deviations of θ_t from its long-run value result initially in movements in the dollar-denominated UK inflation rate but not in the US inflation rate[10]. So, for example, if θ_t exceeds its long-run value (a real undervaluation of the dollar), there is no tendency for Δp_{t+1} to be higher than otherwise. Instead, there is a tendency for a fall in the dollar-denominated UK inflation rate, either because Δp^*_{t+1} or Δe_{t+1} or both tend to be lower than otherwise would be the case. The split into lower UK inflation or a nominal appreciation of the dollar will depend, in part at least, on the prevailing nominal exchange rate regime.

Figure 1 shows the estimated spectral density of Δp, $h_{\Delta p}(\omega)$, indicating that part due to feedback from $\Delta(p^* + e)$, $g_{\Delta(p^*+e)\rightarrow\Delta p}(\omega)$, while Figure 2 shows the estimated spectral density of $\Delta(p^* + e)$, $h_{\Delta(p^*+e)}(\omega)$, indicating that part due to feedback from Δp, $g_{\Delta p\rightarrow\Delta(p^*+e)}(\omega)$. Figure 1 reveals that the feed back from $\Delta(p^* + e)$ to Δp is zero at the zero frequency and accounts for only a small fraction of the feedback at other frequencies. Figure 2 shows that the feedback from Δp to $\Delta(p^* + e)$ accounts for about half the variation in $\Delta(p^* + e)$ at the zero frequency and that most of the feedback is due to that at low frequencies.[11] The feedback at the zero frequency is, of course, responsible for the cointegration of Δp and $\Delta(p^* + e)$ and the stationarity of θ.

Decomposition of the feedback between Δp and $\Delta(p^* + e)$ by frequency provides some insight into the usual results of cointegration approaches to testing the PPP hypothesis using data from the most recent experience with floating exchange rates among the major currencies. As this period spans about 24 years, data from it cannot contain information about any cycles with periods longer than 24 years - those associated with frequencies less

length with probability one tend to select lag lengths shorter than the true length in finite samples. We have chosen to err on the side of selecting lag lengths that are too long rather than too short due to Geweke's (1982) concerns about the effects of inappropriately short lag lengths on the feedback decompositions.

[10]Adding a π_{t-1} term to the Δp_t equation yields an estimated coefficient of $-.009$ with an absolute t-ratio of .15 while not changing any of the selected lag lengths and producing only minor changes in the estimates of the other parameters.

[11]We know of no distribution theory that would enable us to get a sense of the sampling variation in our estimates but the significance of the coefficients in our estimated model implies that the feedback is significantly different from zero for at least some frequencies.

than $\frac{\pi}{12}$[12]. The estimate of $g_{\Delta p \to \Delta(p^*+e)}(\omega)$ in Figure 2 implies that about 60% of the feedback from Δp_t to $\Delta(p_t^* + e_t)$ occurs at such frequencies and about 80% occurs at frequencies less than $\frac{\pi}{6}$ - that part of the variation in the data due to cycles with periods greater than 12 years[13]. In other words, data from the recent float contains little or no information about cycles at those frequencies contributing most of the feedback between the two time series[14].

This claim may seem to be at odds with the sense of the speed of adjustment toward PPP given by the usual half-life calculation. For example, estimating the model $\theta_t = \gamma + \rho\theta_{t-1} + \nu_t$ using the data on θ_t used in the feedback calculations above yields $\widehat{\rho} = .887$ implying that deviations of θ_t from its mean have a half-life of 5.8 years[15]. However, integrating the spectral density of θ_t implied by this estimate shows that about 73% of its variance is due to cycles with periods greater than or equal to 24 years and about 86% is due to cycles with periods greater than or equal to 12 years. As these numbers are consistent with those calculated from the feedback decomposition, the resolution of the two approaches seems to be that the half-life concept severely overstates the speed of return to long-run PPP.

As Geweke's feedback decomposition can be interpreted as the frequency domain counterpart of variance decomposition in time domain (Koo, 1996), the caveats applying to Granger “causality” also apply here. In particular, one variable can Granger-cause another while explaining only a small portion of its variability. Our method should thus be understood as an indirect test of purchasing power parity rather than an attempt to measure the degree of causal relationship between the US and the UK prices.

4. Conclusions

We have shown how to adapt Geweke's (1982) method of feedback decomposition by frequency to the case of $I(1)$ time series generated by a bivariate error–correction model. This case is of interest because, while all cointegrated time series can be represented by an error–correction model, their data generating processes cannot be approximated by a finite VAR as Geweke's method requires. This adaptation is applied to data on US and UK exchange rates and price levels over the period 1791 to 1990. We find that much of the feedback between the two time series occurs at frequencies associated with cycles having periods exceeding 24 years providing some insight into the rejections of the purchasing power parity hypothesis typically found in those studies testing the stationary of the log of

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[12]The period of a cycle with frequency ω is $\frac{2\pi}{\omega}$. As we use annual data, the frequencies π, $\frac{3\pi}{4}$, $\frac{\pi}{2}$, $\frac{\pi}{4}$, and 0 correspond to cycles with periods of 2 years, $2\frac{2}{3}$ years, 4 years, 8 years and infinity respectively.

[13]That is, $\int_0^{\pi/12} g_{\Delta p \to \Delta(p^*+e)}(\omega)d\omega \simeq .6\int_0^{\pi} g_{\Delta p \to \Delta(p^*+e)}(\omega)d\omega$ and $\int_0^{\pi/6} g_{\Delta p \to \Delta(p^*+e)}(\omega)d\omega \simeq .8\int_0^{\pi} g_{\Delta p \to \Delta(p^*+e)}(\omega)d\omega$.

[14] Abuaf amd Jorion (1990) and Frankel (1990), among others, have made the same point.

[15]This is, of course, the same estimate and calculation reported by Lothian and Taylor (1996). The AIC selects the AR(1) specification over all other AR models up to AR(10). For this process, the half-life is defined as n such that $E(\pi_{t+n} - E\pi_t|\pi_t) = \frac{1}{2}(\pi_t - E\pi_t)$. As $E(\pi_{t+n} - E\pi_t|\pi_t) = \rho^n(\pi_t - E\pi_t)$, $\rho^n = \frac{1}{2}$ or $n = -\frac{\log 2}{\log \rho}$. In a study using GLS estimation on real exchange rate data from eight industrialized countries relative to the US over the 1901 to 1972 period, Abuaf and Jorion (1990) report an estimate implying a half-life of 3.7 years. Frankel (1990) uses data on the US and UK price levels and exchange rates over the 1869 to 1987 and reports an estimate implying a half-life of 4.1 years.

the real exchange rate using data from the most recent experience with floating exchange rates.

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Figure 1: Spectral Density of Δp and Feedback from $\Delta(p^{*}+e)$

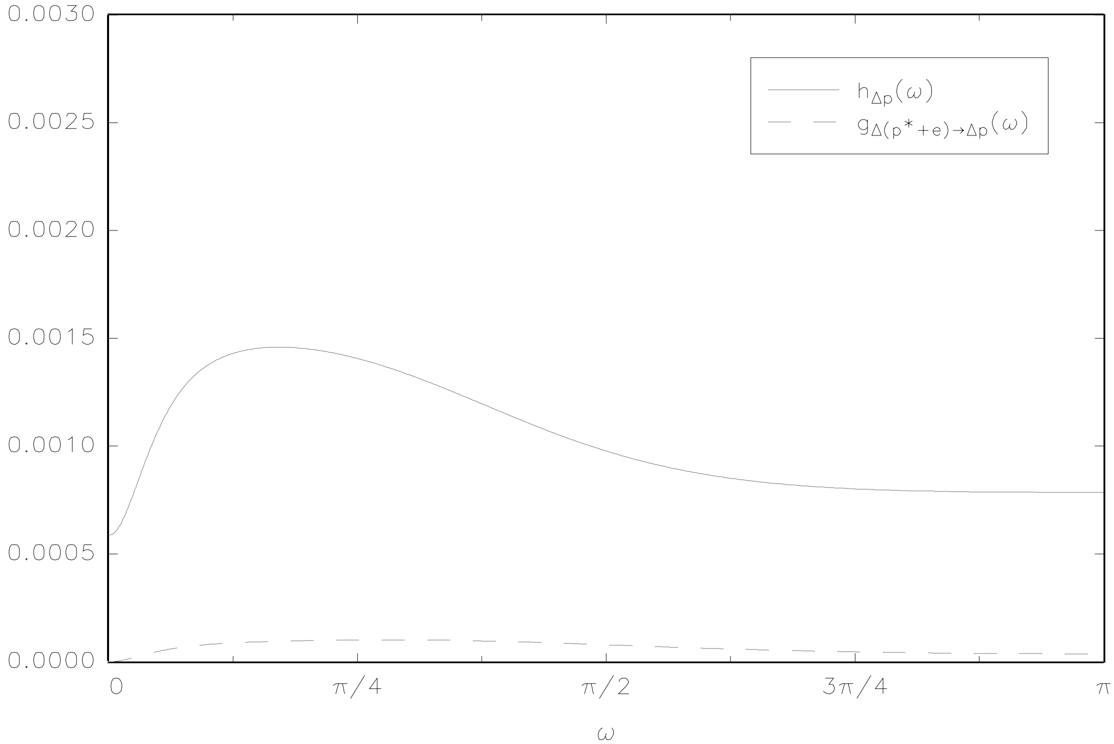

Figure 2: Spectral Density of $\Delta(p^{*}+e)$ and Feedback from Δp